Int. J. Heat Mass Transfer. Vol. 25, No. 8 pp. 1245–1247, 1982
Printed in Great Britain

0017-9310/82/081245-02 $03.00/0

SHORTER COMMUNICATIONS

MIXED CONVECTION ABOUT A HORIZONTAL CYLINDER AND A SPHERE IN A FLUID-SATURATED POROUS MEDIUM

P. Cheng
Department of Mechanical Engineering, University of Hawaii at Manoa,
Honolulu, HI 96822, U.S.A.

(*Received* 12 *November* 1981 *and in final form* 18 *January* 1982)

NOMENCLATURE

A_n, constant defined in equation (5), $A_0 = 2$ for a cylinder and $A_1 = 3/2$ for a sphere;
f, dimensionless stream function defined in equation (6);
G_n, function defined in equation (6);
Gr, Grashof number, $Gr = (T_w - T_\infty)Kg\beta L/v^2$;
g, gravitational acceleration;
H_n, function defined in equation (8);
h, local heat transfer coefficient;
K, permeability of the porous medium;
k, thermal conductivity of the saturated porous medium;
n, constant in equations (1) and (2), $n = 0$ for a cylinder and $n = 1$ for a sphere;
Nu_x, local Nusselt number, $Nu_x = hx/k$;
Pe_x, local Peclet number, $Pe_x = U_\infty x/\alpha$;
q_w, surface heat flux;
r, radial distance from the symmetrical axis to the surface of a sphere;
r_0, radius of a cylinder or a sphere;
Re, Reynolds number, $Re = U_\infty L/v$;
T, temperature;
U_∞, velocity at infinity;
u, Darcy's velocity, x-direction;
v, Darcy's velocity, y-direction;
x, coordinate measured in the streamwise direction along the surface of a cylinder or sphere from the lowest point;
y, coordinate perpendicular to the surface of a cylinder or a sphere.

Greek symbols
α, equivalent thermal diffusivity;
β, the coefficient of thermal expansion;
δ, boundary layer thickness;
η, similarity variable;
θ, dimensionless temperature;
μ, viscosity of the convective fluid;
v, kinematic viscosity of the convective fluid;
ρ, density of the convective fluid;
ϕ, angle measured from the downward vertical to the y-axis;
ψ, stream function;
χ, dimensionless distance in the x-direction.

Subscripts
∞, condition at infinity;
w, condition at the wall.

INTRODUCTION

Convection of groundwater about hot intrusives can be modelled as heated bodies embedded in a fluid-saturated porous medium. Based on boundary layer approximation, Cheng [1] has obtained similarity solutions for mixed convection about a heated vertical plate in a porous medium at high Peclet numbers. In this paper, the problems of mixed convection about a horizontal isothermal cylinder and an isothermal sphere are considered. It is shown that with a generalized similarity transformation (similar to those employed previously by Merkin [2] for another problem), the resulting ordinary differential equations and boundary conditions for the present problems reduce to those of a vertical isothermal surface in a porous medium.

ANALYSIS

The physical problems under consideration are shown in Fig. 1 where x is the coordinate in the streamwise direction along the surface of the cylinder or a sphere from the lowest point, and y is the coordinate perpendicular to the surface; r_0 is the radius of the cylinder or the sphere and $\phi = x/r_0$ is the angle of the y-direction with respect to the vertical; T_w and T_∞ are the temperatures of the heated surface and at infinity respectively. If the thickness of the boundary layer δ is thin such that $\delta \ll r_0$, the boundary layer approximation is applicable. With such an approximation and the assumption that the buoyancy force normal to the heated surface is negligible, the governing equations for mixed convection about a horizontal cylinder and a sphere are

$$\frac{1}{r^n}\frac{\partial^2\psi}{\partial y^2} = \frac{K\rho_\infty\beta g}{\mu}\sin(x/r_0)\frac{\partial T}{\partial y} \tag{1}$$

$$\alpha\frac{\partial^2 T}{\partial y^2} = \frac{1}{r^n}\left[\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}\right] \tag{2}$$

where the stream function ψ is defined as

$$u = \frac{1}{r^n}\frac{\partial\psi}{\partial y} \quad \text{and} \quad v = -\frac{1}{r^n}\frac{\partial\psi}{\partial x}$$

with u and v denoting the Darcian velocities in the x and y directions, and $n = 0$ for a horizontal cylinder and $n = 1$ for a sphere; $r = r_0\sin(x/r_0)$; ρ, μ and β are the density, viscosity and the thermal expansion coefficient of the fluid; K is the intrinsic permeability of the porous medium; α is the thermal diffusivity of the saturated porous medium; and g is the gravitational acceleration. Boundary conditions at the wall and at infinity are

$$y = 0: \quad \frac{\partial\psi}{\partial x} = 0, \qquad T = T_w \tag{3}$$

$$y \to \infty: \frac{1}{r^n}\frac{\partial\psi}{\partial y} \to U(x), \qquad T = T_\infty \tag{4}$$

where $U(x)$ is the Darcian velocity in the tangential direction outside of the boundary layer. From the potential theory, the

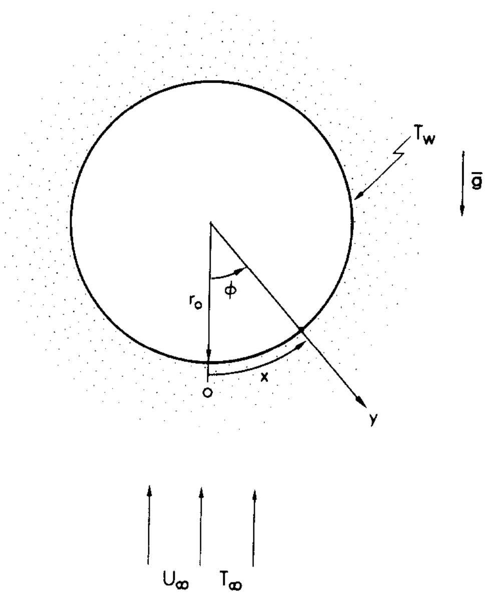

FIG. 1. The coordinate system for mixed convection about a horizontal cylinder and a sphere in a porous medium.

explicit expression for $U(x)$ on the surface of the cylinder and the sphere is

$$U(x) = U_\infty A_n \sin(x/r_0) \tag{5}$$

where $A_0 = 2$ and $A_1 = 3/2$ for the cylinder and the sphere respectively.

We now introduce the following similarity transformations:

$$\psi = \alpha r_0^n (A_n U_\infty r_0/\alpha)^{1/2} G_n(\chi) f(\eta) \tag{6}$$

$$\theta = (T - T_\infty)/(T_w - T_\infty) \tag{7}$$

$$\eta = (A_n U_\infty r_0/\alpha)^{1/2} (y/r_0) H_n(\chi) \tag{8}$$

where $\chi = x/r_0$

$$G_0(\chi) = (1 - \cos\chi)^{1/2},\quad H_0(\chi) = \sin\chi/G_0(\chi),$$

$$G_1(\chi) = \left[\frac{\cos^3\chi}{3} - \cos\chi + \frac{2}{3}\right]^{1/2},$$

and

$$H_1(\chi) = \sin^2\chi/G_1(\chi).$$

Substituting equations (6)–(8) into equations (1)–(4) yields

$$f'' = Gr/Re\,\theta' \tag{9}$$

$$\theta'' = -f\theta'/2 \tag{10}$$

$$f(0) = 0,\quad \theta(0) = 1 \tag{11a, b}$$

$$f'(\infty) = 1,\quad \theta(\infty) = 0 \tag{12a, b}$$

where the primes are the differentiation with respect to η; $Gr/Re = Kg\beta g(T_w - T_\infty)/\mu U_\infty A_n$ is the ratio of the Grashof number to the Reynolds number based on any length, and is a dimensionless parameter measuring the relative importance of the free to forced convection. Equations (9) and (10) subject to boundary conditions (11) and (12) have been solved numerically by Cheng [1] for the problem of mixed convection about a vertical isothermal heated surface in a porous medium.

RESULTS AND DISCUSSION

The Darcian velocities in terms of the transformed variables are

$$u = U_\infty A_n \sin\chi f'(\eta) \tag{13a}$$

and

$$v = -\frac{(A_n U_\infty \alpha/r_0)^{1/2}}{\sin^n\chi}\left[\frac{G_n(\chi)H_n'(\chi)\eta f'(\eta)}{H_n(\chi)} + G_n'(\chi)f(\eta)\right] \tag{13b}$$

where $H_n'(\chi) = \mathrm{d}H_n(\chi)/\mathrm{d}\chi$ and $G_n'(\chi) = \mathrm{d}G_n(\chi)/\mathrm{d}\chi$. The local surface heat is given by

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$= k(T_w - T_\infty)[A_n U_\infty/(\alpha r_0)]^{1/2} H_n(\chi)[-\theta'(0)] \tag{14}$$

where k is the thermal conductivity of the porous medium. Equation (14) can be expressed in dimensionless form as

$$\frac{Nu_x}{Pe_x^{1/2}} = (A_n\chi)^{1/2} H_n(\chi)[-\theta'(0)] \tag{15}$$

where $Nu_x = q_w x/k(T_w - T_\infty)$ and $Pe_x = U_\infty x/\alpha$ are the local Nusselt number and the local Peclet number respectively.

The value of $[-\theta'(0)]$ as a function of Gr/Re is given in a previous paper [1].

REFERENCES

1. P. Cheng, Combined free and forced convection flow about inclined surfaces in porous media, *Int. J. Heat Mass Transfer* **20**, 807–814 (1977).
2. J. H. Merkin, Free convection boundary layers on axisymmetric and two-dimensional bodies of arbitrary shape in a saturated porous medium, *Int. J. Heat Mass Transfer* **22**, 1461–1462 (1979).